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Running of gauge couplings in AdS_5 via deconstruction

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Abstract: Running of gauge couplings on a slice of AdS_5 is examined using the deconstruction setup. Logarithmic running instead of (linear) power law is justified when the cutoff is lower than the curvature scale. Most of interesting features including the localization of Kaluza-Klein modes and the widening of higher Kaluza-Klein spectrum spacing are well captured within the framework of deconstruction.

Keywords: Renormalization Group, Field Theories in Higher Dimensions, GUT.

Contents

1. Introduction

In recent years new solutions to the hierarchy problem based on the presence of extra space-time dimensions have been proposed. In refs. [1, 2] the weakness of gravity compared to the gauge interactions was explained with the aid of large volume of flat extra dimensions in which only gravitons propagate. On the other hand, if the space is curved along the extra dimension, one can achieve a large hierarchy even for a small size of the extra dimension [3]. The setup proposed by Randall and Sundrum involved a slice of AdS_5 space-time terminated by two branes: the Planck brane and the TeV brane where the Standard Model fields were assumed to live. The hierarchy appears due to the presence of a large warp factor which suppresses all the mass scales (in particular the fundamental (cutoff) scale) on the TeV brane. At the same time, the gravitational interactions are weak since the massless graviton mode has a small overlap of the wave function with the TeV brane matter.

Randall and Sundrum proposed this setup as an alternative to the MSSM or the large extra dimensions. However, the question remains whether the model can be reconciled with the gauge coupling unification as the low-energy effective theory breaks down close to TeV energies and quantum gravity effects emerge. There seems no chance to embed in the Randall-Sundrum model the triumph of gauge coupling unification in the MSSM since there is no huge scale difference which allows three different couplings to be unified with large log corrections.

However recently it has been realized that the Randall-Sundrum model can be made compatible with the large-scale unification if the Standard Model gauge fields are allowed to propagate in the bulk AdS_5 space-time. The question of the running of gauge couplings in AdS_5 was first studied by Pomarol [4] who showed, that if the cutoff of the theory is below the curvature scale of AdS_5, logarithmic running occurs instead of linearly divergent power law which happens in flat 5D. In Pauli-Villars regularization all the heavy Kaluza-Klein modes are paired up with the modes of the regulating ones and thus they give a negligible contribution for the running. The large log correction arises due to the differences of the infrared(IR) cutoff scale μ and the ultraviolet(UV) cutoff given by the regulator mass. Randall and Schwartz [5, 6] used a different prescription inspired by AdS/CFT correspondence in order to show the logarithmic running of the bulk gauge couplings. A position dependent momentum cutoff was used as a regularization and, as a consequence, the gauge coupling got logarithmic corrections as long as the cutoff was below the scale of the curvature. Choi, Kim and Kim [7] derived general running equations for the gauge couplings in an orbifold of AdS_5 in the supersymmetric setup based on holomorphy. Dimensional regularization [8, 9] shows the momentum dependence ($\log p$), the radius dependence ($\log R$) and the divergent structure ($\log \Lambda$ or $\log \mu$) separately. Following this approach, one can derive all the radius dependence and the momentum dependence of the gauge couplings by reading the Kahler metric and considering possible threshold corrections in the presence of the curvature. Radius running [9] is a nice way to reach the coupling defined at UV. In [10], more detailed analysis was done using the Pauli-Villars regularization and it confirmed the results of Pomarol. Holographic evolution of gauge couplings was discussed in [11].

In this paper we approach the problem using the deconstruction setup. In refs. [12, 13] it was observed that gauge theories in four dimensions with the gauge group $\mathrm{SU}(N_c)^N$ and scalars (link-Higgs fields) in bifundamental representations appear to be equivalent to a higher dimensional $\mathrm{SU}(N_c)$ theory in the flat background.[1] The correspondence holds below some deconstruction scale v set by the expectation values of the link-Higgs fields that break the group $\mathrm{SU}(N_c)^N$ to its diagonal subgroup. Below v the spectrum and interactions of the four dimensional degrees of freedom are analogous to that of the Kaluza-Klein(KK) modes of gauge bosons propagating in extra dimensions. Above the scale v, a deconstruction model can be considered as a (possibly renormalizable) UV completion of the corresponding extra dimensional theory. Hence, there appeared a new possibility to study the extra dimensional physics using the standard tools of four dimensional Yang-Mills theories. Recently it has been noticed that also gauge fields propagating in a curved background can be described by four dimensional gauge theories [17] (see also [18]). This can be simply achieved using the analogous set-up as that of [12, 13] but with non-universal vevs of the link-Higgs fields. Thus, deconstruction can also help to understand the physics of the curved space.

Deconstruction is used in our paper as an alternative regularization method and is compared to other regularizations. We show that the Renormalization Group(RG) running equation is directly related to the boundary condition for the gauge couplings that we start with, and is highly dependent on the UV completion. We restrict the study to the region in which the cutoff scale Λ (the inverse of the lattice size) is lower than the curvature scale k ($\Lambda \ll k$) though a more general study is possible with the aid of numerical analysis for $\Lambda \sim k$. The other limit (the cutoff scale is much higher than the curvature scale, $\Lambda \gg k$) is very similar to the flat space since we can not feel the presence of the curvature for $\Lambda \gg k$. There are two interesting observations for $\Lambda \ll k$. Firstly, it is shown that in deconstruction there are less heavy states than in the low energy effective description of

[1]Similar models had been studied previously for different reasons, see [14, 15, 16].

the Randall-Sundrum(RS) model and the number of KK modes increases logarithmically rather than linearly in energy scale. This makes the running of the gauge coupling soften compared to the flat case and the rather mild running instead of power law running is expected. Secondly, the running of the gauge coupling above TeV is not a running of the zero mode but rather a running of the part of the gauge coupling existing near Planck brane. This is related to the definition of the Wilsonian gauge coupling from which we can find the relation with the low energy coupling measured by experiment. The natural Wilsonian gauge coupling at Λ is defined at different scales for different positions along the fifth dimension (which is Λ for Planck brane and is $\Lambda e^{-\pi k R}$ for TeV brane), and we can not apply the zero mode running formula used in the flat space extra dimension. This reduces the effective number of KK modes at a given energy scale from $\log E$ to 1, and we get the running formula which is just the same as the four dimensional one.

2. Setup

In this section we summarize the correspondence between the 5D gauge theories and the deconstruction models. In a warped background with the metric $ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ the 5D Yang-Mills action reads:

$$\delta S_{5D} = \int d^4x \int_0^{\pi R} dy \, \mathrm{Tr}\left(-\frac{1}{2}F_{\mu\nu}F_{\mu\nu} + a^2(y)F_{\mu 5}F_{\mu 5}\right), \tag{2.1}$$

where the field strength is defined as $F_{\alpha\beta} = 2\frac{1}{g_5}\partial_{[\alpha}(g_5 A_{\beta]}) + ig_5[A_\alpha, A_\beta]$ and g_5 is the gauge coupling (here we allow g_5 to be position dependent). One can discretize the compact fifth dimension into N equally sized intervals (y_i, y_{i+1}), with the lattice spacing $\Delta y \equiv y_{i+1} - y_i = R/N$. The integral over the y coordinate is traded for the sum, $\int dy \to \sum_{i=1}^{N} \Delta y$, and the derivates along the fifth dimension are discretized as $\partial_5 A_\mu|_{y_i} \to \frac{A_{\mu,i+1} - A_{\mu,i}}{\Delta y}$.

The deconstruction models which mimic the 5D SU(N_c) gauge theories compactified on the orbifold S_1/Z_2 involve N copies of SU(N_c) group with the gauge couplings g_i $(i = 1, \ldots, N)$ and link Higgs fields Q_i $(i = 1, \ldots, N-1)$. The links are bifundamental under SU$(N_c)_i \times$SU$(N_c)_{i+1}$, thus they are represented by $N_c \times N_c$ complex matrices. The action is:

$$S_{4D} = \int d^4x \sum_{i=1}^{N} \mathrm{Tr}\left(-\frac{1}{2}F_{\mu\nu,i}F_{\mu\nu,i} + 2D_\mu Q_i^\dagger D_\mu Q_i + V(Q_i)\right). \tag{2.2}$$

The link-Higgs field can be conveniently split as $Q_i = \frac{1}{\sqrt{2}}(\Sigma_i + iG_i)$. The scalar potential $V(Q_i)$ is necessary go give the link-Higgs fields the desired pattern of vacuum expectation values, but the details of it are not relevant for us.

When the link-Higgs bosons acquire vacuum expectation values, $\langle Q_i \rangle = v_i$, the full gauge symmetry is broken down to the diagonal subgroup and the usual Higgs mechanism gives masses to the gauge fields corresponding to broken generators. The action then reads:

$$\begin{aligned} S_{4D} &= \int d^4x \sum_{i=1}^{N} \mathrm{Tr}\left(-\frac{1}{2}F_{\mu\nu,i}F_{\mu\nu,i} + Z_{\mu,i}^\dagger Z_{\mu,i}\right), \\ Z_{\mu,i} &= (\partial_\mu G_i - i\partial_\mu \Sigma_i + v_i(g_i A_{\mu,i} - g_{i+1}A_{\mu,i+1}) + \\ &\quad + g_i A_{\mu,i}(\Sigma_i + iG_i) - (\Sigma_i + iG_i)g_{i+1}A_{\mu,i+1}). \end{aligned} \tag{2.3}$$

In non-supersymmetric models the fields Σ_i have no interpretation in terms of 5D degrees of freedom.[2] Thus they should be given a large mass (by adjusting the scalar potential $V(Q_i)$ appropriately) and decoupled from the low-energy theory. In the following we assume that this step has been done and neglect Σ in our analysis. The remaining terms in $Z_{\mu,i}$ can be arranged as follows:

$$Z_{\mu,i} = \left(\partial_\mu G_i + v_i g_i \frac{g_i A_{\mu,i} - g_{i+1} A_{\mu,i+1}}{g_i} + i g_i \left[A_{\mu,i}, G_i\right] - i G_i \left(g_{i+1} A_{\mu,i+1} - g_i A_{\mu,i}\right)\right) . \quad (2.4)$$

Now we can do the following identification between the degrees of freedom in our deconstruction model and the 5D gauge theory:

$$\begin{aligned} g_5(y_i)\frac{1}{\sqrt{\Delta y}} &\longrightarrow g_i\,, \\ A_\mu(y_i)\sqrt{\Delta y} &\longrightarrow A_{\mu,i}\,, \\ A_5(y_i)\sqrt{\Delta y} &\longrightarrow G_i\,, \\ \frac{a(y_i)}{\Delta y} &\longrightarrow g_i v_i\,. \end{aligned} \quad (2.5)$$

The problem at what scale should this matching actually be done will be discussed later on. The correspondence between the 4D part of the field strengths $F_{\mu\nu}^2$ in the two theories is obvious. The other term in (2.3) becomes:

$$Z_{\mu,i}^\dagger Z_{\mu,i} = \Delta y a(y)^2 \left|\partial_\mu A_5 - \frac{1}{g_5}\partial_5(g_5 A_\mu) + i g_5 [A_\mu, A_5] - i\Delta y A_5 \partial_5 (g_5 A_\mu)\right|^2 |_{y_i} \quad (2.6)$$

and we recover the $F_{\mu 5}^2$ term of the discretized 5D action (2.1) up to the $\mathrm{O}(\Delta y^2)$ correction. In 5D interpretation this correction can be considered as a higher-derivative term suppresed by the appropriate power of the fundamental scale. Thus, in the deconstruction models, the role of the fundamental scale is played by the inverse of the lattice spacing, $\Lambda \sim (\Delta y)^{-1}$.

For $v_i = v$, $g_i = g$ the deconstruction model describes, at low energies, the same physics as that of the 5D gauge theory in the flat background [12, 13]. In this case it is straightforward to diagonalize the mass matrix for the gauge bosons and the spectrum turns out to be

$$m_n = 2gv \sin\left(\frac{n\pi}{2N}\right), \qquad (n = 0, \ldots, N-1)\,. \quad (2.7)$$

For $n \ll N$, we obtain the well-known spectrum of Kaluza-Klein modes of a 5D gauge theory compactified on S^1/Z_2, $m_n = n/R$, with the radius $R = N/\pi g v$. Thus there are three distinctive energy scales. Below the scale $gv\pi/N$ the theory is analogous to the ordianary 4D gauge theory. In the range from $gv\pi/N$ up to the inverse lattice spacing gv the heavy gauge bosons can be seen and the physics is analogous to that of the 5D gauge theory. Finally, above gv the theory is an unbroken 4D gauge theory with the product

[2] In supersymmetric models they are matched to the real scalar component of the 5D $\mathcal{N} = 2$ gauge multiplet [19].

group $\mathrm{SU}(N_c)^N$. In this energy range the deconstruction model can be considered as one of many possible UV completions of the 5D gauge theory with the cut-off $\Lambda = gv$. Running of the gauge couplings in "flat" deconstruction models was studied in [20, 21, 22].

It was noticed in [17] that if the quantity $g_i v_i$ depends on i, such models correspond to 5D gauge theories with a non-trivial warp factor $a(y)$. In particular choosing $g_i = g$ and $v_i = v\varepsilon^i$, $\varepsilon^N \approx 10^{-16}$, we are able to describe the physics of gauge fields (with a constant gauge coupling) living in the Randall-Sundrum background. One can understand it by noting, that $g_i v_i$ plays the role of an effective lattice spacing measured by the warped metric. In consequence, the scale at which we switch from 5D description to the UV completion depends on the position in the group space. This is similar in spirit to the notion of a position dependent momentum cut-off introduced in [5].

Using eqs. (2.5) one finds the connection between the parameters of the two theories:

$$\begin{aligned}
\frac{1}{gv_1} &\longrightarrow \Delta y \sim \frac{1}{\Lambda}\,, \\
\frac{1}{gv_i} &\longrightarrow \Delta y \sim \frac{1}{\Lambda e^{-ky}}\,, \qquad \text{(for 4D observer)}\,, \\
\frac{N}{\pi g v_1} &\longrightarrow R\,, \qquad \left(\varepsilon^N \to e^{-k\pi R}\right), \\
gv_1 \log\left(\frac{1}{\varepsilon}\right) &\longrightarrow k\,, \\
gv_{N-1} &\longrightarrow \Lambda e^{-k\pi R}\,, \\
\varepsilon &\longrightarrow e^{-k/\Lambda}\,, \qquad (\varepsilon \ll 1 \to \Lambda \ll k)\,.
\end{aligned} \tag{2.8}$$

The mass matrix of the gauge bosons is:

$$2g^2v_1^2 \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & \cdots & 0 \\
-1 & 1+\varepsilon^2 & -\varepsilon^2 & 0 & 0 & \cdots & 0 \\
0 & -\varepsilon^2 & \varepsilon^2+\varepsilon^4 & -\varepsilon^4 & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & 0 & -\varepsilon^{2(N-3)} & \varepsilon^{2(N-3)}+\varepsilon^{2(N-2)} & -\varepsilon^{2(N-2)} \\
0 & \cdots & 0 & 0 & 0 & -\varepsilon^{2(N-2)} & \varepsilon^{2(N-2)}
\end{bmatrix}. \tag{2.9}$$

Unfortunately, for $\varepsilon \neq 1$, it is difficult to diagonalize the mass matrix and the eigenvalues must be obtained numerically. However, for $\varepsilon \ll 1$, we can diagonalize the mass matrix step by step and the result remains trustworthy. Therefore, from now on, the case with $\varepsilon \ll 1$ is considered. In such case the following simple picture holds. At the scale v_1 the first two groups $\mathrm{SU}(N_c)_1 \times \mathrm{SU}(N_c)_2$ are broken to the diagonal group $\mathrm{SU}(N_c)_{(2)}$, at the scale v_2 $\mathrm{SU}(N_c)_{(2)} \times \mathrm{SU}(N_c)_3$ breaks to $\mathrm{SU}(N_c)_{(3)}$ and so on. We use the notation $\mathrm{SU}(N_c)_i$ for the i-th gauge group of the original product, $\mathrm{SU}(N_c)_{(i)}$ for the unbroken gauge group after the Higgsing of $\mathrm{SU}(N_c)_{(i-1)} \times \mathrm{SU}(N_c)_i$ and we denote $g_{(i)}$ the corresponding gauge coupling. Figure 1 shows the moose diagram.

Thus, for $\varepsilon \ll 1$, the problem is reduced to diagonalizing 2×2 matrices and the eigenvalues are easily determined. First we diagonalize the part involving $A_{\mu,1}$ and $A_{\mu,2}$ and the eigenvalues are $\mathrm{O}(gv\varepsilon)$ and $\mathrm{O}(gv\varepsilon^2)$. One state with $\mathrm{O}(gv\varepsilon)$ eigenvalue is isolated

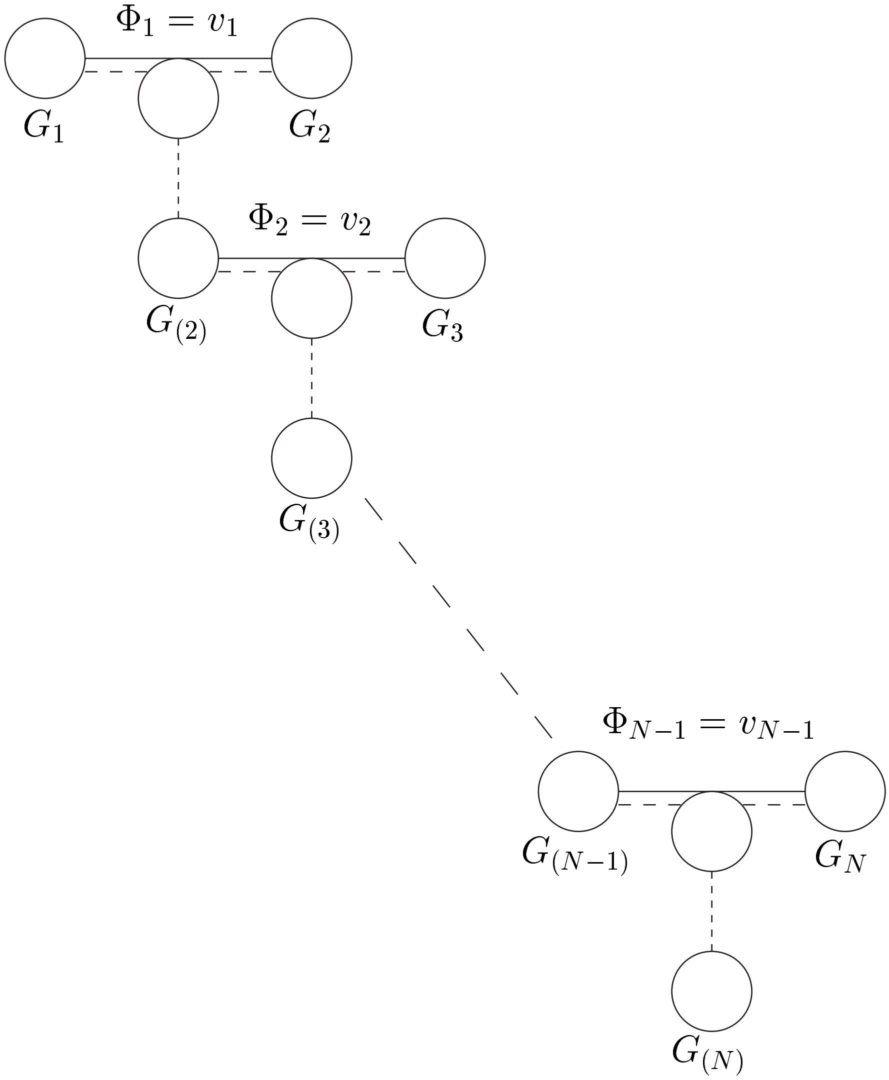

Figure 1: Moose diagram for warped gauge theory.

and remains as a true eigenstate while the other with $\mathrm{O}(gv\varepsilon^2)$ has a mixing term with $A_{\mu,3}$. We can do the same procedure iteratively and finally we obtain the spectrum $\mathrm{O}(gv\varepsilon^{N-1})$ corresponding to the final Higgsing scale. Exact calculation shows that there is always a zero-mode eigenstate corresponding to the full diagonal subgroup. This is easily seen from the fact that the determinant of the original mass matrix is zero or by finding the zero mode eigenstate given as

$$A_\mu^{(0)} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} A_{\mu,i} \,, \tag{2.10}$$

which reflects that the zero mode of the vector field is constant along the extra dimension in AdS_5 orbifold. For other higher mode eigenvalues, the eigenstates are given as[3]

$$\begin{aligned}
A_\mu^{(N-1)} &= \frac{1}{\sqrt{2}}(A_{\mu,1} - A_{\mu,2}), \qquad \text{up to } \mathrm{O}(\varepsilon^2), \\
A_\mu^{(N-2)} &= \frac{1}{\sqrt{6}}(A_{\mu,1} + A_{\mu,2} - 2A_{\mu,3}), \qquad \text{up to } \mathrm{O}(\varepsilon^2), \\
&\cdots \\
A_\mu^{(N-j)} &= \frac{1}{\sqrt{j(j+1)}}(A_{\mu,1} + \cdots + A_{\mu,j} - jA_{\mu,j+1}), \qquad \text{up to } \mathrm{O}(\varepsilon^2), \\
&\cdots \\
A_\mu^{(1)} &= \frac{1}{\sqrt{(N-1)N}}(A_{\mu,1} + \cdots + A_{\mu,N-1} - (N-1)A_{\mu,N}), \qquad \text{up to } \mathrm{O}(\varepsilon^2).
\end{aligned} \tag{2.11}$$

$A_\mu^{(N-j)}$ has an eigenvalue of order $\mathrm{O}(v\varepsilon^j)$ and the 5D interpretation is clear. For higher modes ($m_n \sim v$), the corresponding wave functions are localized at the Planck brane. This is exactly the position dependent momentum cutoff for the gauge fields which Randall and Schwartz introduced for the calculation of the gauge coupling running in AdS_5 orbifold [5, 6]. Thus, the position dependent momentum cutoff is naturally realized in the deconstruction setup. For the flat extra dimension ($\varepsilon = 1$), the lowest modes have fewer nodes since they have smaller gradients and give lighter Kaluza-Klein states. However, if the extra dimension is highly warped, the scale felt by the mode is different and the lowest excited mode is the one that has a variation at very near the TeV brane. In our analysis $\varepsilon \ll 1$ and, using eqs. (2.8), this corresponds to $k \gg \Lambda$ which means the curvature effect is crucial. If the cutoff is much lower the curvature scale, then the net degree of freedom encountered as the energy increases is less than in the flat case as the Kaluza-Klein level spacing becomes larger. This is the momentum space version of the position dependent momentum cutoff and is also related to the entropy counting of black holes which is proportional to the area rather than the volume [23].

For large N, $\varepsilon \sim 1$ even for the Randall-Sundrum geometry and the second lightest mode would have two nodes near the TeV brane, etc. As $\varepsilon \sim 1$ corresponds to $k \ll \Lambda$, the cutoff is much above the curvature scale. Then above the curvature scale, we start to see the flat five dimensional physics. Unfortunately this case is beyond the scope of our analysis since the nice separation of scales is not allowed and only numerical works can give results.

3. Running of gauge couplings

The running behaviour of the gauge couplings in the AdS_5 background is an important question both from the theoretical and the phenomenological point of view. However, different regularizations show different results which do not exactly coincide. In this section

[3]The expression contains a term up to $\mathrm{O}(\varepsilon^2)$. Here we assume $\varepsilon \ll 1$. However, even at $\mathrm{O}(\varepsilon^2)$ there can be order one or bigger than order one corrections due to N which can be extremely large. $N\varepsilon^2$ can not be neglected with simple assumption $\varepsilon \ll 1$. Therefore, it is assumed also that $N\varepsilon^2 \ll 1$ such that the potentially dangerous correction can be neglected. In the realistic case explaining the gauge hierarchy, we need $\varepsilon^N = 10^{-16}$. The choice of $\varepsilon = 1/10$ and $N = 16$ satisfies the second assumption $N\varepsilon^2 = 0.16 \ll 1$.

we study this problem using the decontruction setup described in the previous section. We concentrate on the case where only the gauge and the link-Higgs fields are replicated (live in the "bulk") and the matter fields live on either the first or the N-th site, corresponding to matter fields on the Planck and the TeV brane, respectively. The matter fields propagating in the "bulk" are investigated in appendix A.

We find it convenient to study the running using the bottom-up approach. We start with a gauge coupling measured at some low energy scale M_Z, $\alpha(M_Z)$. The running up to the scale of the lowest link-Higgs vev $v\epsilon^{N-1}$ proceeds as in an ordinary 4D gauge theory:

$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha(M_Z)} - \frac{b_0}{2\pi}\log\left(\frac{Q}{M_Z}\right), \qquad Q < v\epsilon^{N-1} \tag{3.1}$$

and b_0 is the beta-function coefficients which gets contributions from the (massless) $\mathrm{SU}(N_c)$ gauge fields and, eventually, from any matter fields which are effectively massless at the considered scale.

At the scale $v\epsilon^{N-1}$ this coupling is identified with the coupling $\alpha_{(N)}$ of the diagonal group. In general, the matching equation at the n-th threshold is:

$$\frac{1}{\alpha_{(n+1)}(v\varepsilon^n)} = \frac{1}{\alpha_{(n)}(v\varepsilon^n)} + \frac{1}{\alpha_{n+1}(v\varepsilon^n)}, \tag{3.2}$$

where α_n is the coupling of the n-th group in the product $\mathrm{SU}(N_c)^N$. For $\varepsilon \ll 1$, as explained in the previous section, the heavy states of gauge bosons can be considered as decoupled between the thresholds. Also, we assumed that the physical links Σ_i are decoupled, and G_i are Goldstone bosons which become longitudinal components of the heavy gauge bosons. Thus the running is given by

$$\frac{1}{\alpha_{(n+1)}(Q)} = \frac{1}{\alpha_{(n+1)}(v\varepsilon^n)} - \frac{b_{(n+1)}}{2\pi}\log\left(\frac{Q}{v\varepsilon^n}\right), \qquad v\varepsilon^n < Q < v\varepsilon^{n+1} \tag{3.3}$$

and the beta-function coefficients $b_{(n+1)}$ include contributions only from the *massless* $\mathrm{SU}(N_c)$ gauge fields and from the matter localized from the first up to the n-th site. Since we assumed that all the matter fields live either on the N-th or on the first site we can write $b_n \equiv b$. (but still $b \neq b_0$ if some matter fields live on the N-th site).

Putting together all these equations, the low-energy gauge coupling $\alpha(M_Z)$ depends on the high-energy couplings in the following way:

$$\begin{aligned}\frac{1}{\alpha(M_Z)} &= \frac{1}{\alpha_1(v)} + \frac{1}{\alpha_2(v\varepsilon)} + \frac{1}{\alpha_3(v\varepsilon^2)} + \cdots + \\ &\quad + \frac{1}{\alpha_N(v\varepsilon^{N-1})} + \frac{b}{2\pi}\log\left(\frac{1}{\varepsilon^{N-1}}\right) + \frac{b_0}{2\pi}\log\left(\frac{v\varepsilon^{N-1}}{M_Z}\right).\end{aligned} \tag{3.4}$$

At this point there seem to be two plausible options for choosing the boundary conditions for the gauge couplings of the product group α_n. One is to choose all the gauge couplings to be equal at the highest scale v. The other is to choose each gauge coupling of the product group so that they have the same value at different scales, namely at the scale where the given group gets broken by the link-Higgs vev. The origin of this ambiguity comes from the question, at which scale the matching between 4D and 5D degrees of freedom should be performed. The second option is motivated by AdS/CFT correspondence of the Randall-Sundrum setup that the cutoff at different positions along the curved extra dimension should be different. In the folowing we investigate the consequences of both approaches.

The first option consist in choosing the boundary conditions of the gauge couplings as

$$\frac{2\pi}{\alpha_1}|_v = \frac{2\pi}{\alpha_2}|_v = \cdots \frac{2\pi}{\alpha_N}|_v = \frac{2\pi}{\alpha}. \tag{3.5}$$

The RG running of each gauge coupling down to the scale at which the corresponding group gets broken is

$$\frac{1}{\alpha_i(v_{i-1})} = \frac{1}{\alpha_i(v)} + \frac{\tilde{b}}{2\pi}\log\left(\frac{v}{v_{i-1}}\right) = \frac{1}{\alpha} + (i-1)\frac{\tilde{b}}{2\pi}\log\left(\frac{1}{\varepsilon}\right). \tag{3.6}$$

Here the beta function coefficient $\tilde{b}$ contains, except for the contribution of the massless gauge fields, also the contribution from the link-Higgs degrees of freedom G and Σ_i (depending on the details of the scalar potential). ¿From the point of view of the 5D RS model this part is highly dependent on the UV completion.

The low energy gauge coupling is:

$$\begin{aligned}\frac{1}{\alpha(M_Z)} &= \frac{N}{\alpha} + (N-1)\frac{b}{2\pi}\log\left(\frac{1}{\varepsilon}\right) + \frac{b_0}{2\pi}\log\left(\frac{v\varepsilon^{N-1}}{M_Z}\right) + \\ &+ \frac{(N-1)(N-2)}{2}\frac{\tilde{b}}{2\pi}\log\left(\frac{1}{\varepsilon}\right). \end{aligned} \tag{3.7}$$

For large N the above formula becomes

$$\frac{1}{\alpha(M_Z)} = \frac{N}{\alpha} + \frac{N^2}{2}\frac{\tilde{b}}{2\pi}\log\left(\frac{1}{\varepsilon}\right). \tag{3.8}$$

Using the dictionary given in eqs. (2.8) (in particular $\log(1/\varepsilon) \to k/\Lambda$ and $N\log(1/\varepsilon) = \log(1/\varepsilon^N) \to \log(\Lambda/\mu)$) the corresponding equation in the 5D interpretation is

$$\frac{1}{\alpha(\mu)} \simeq \frac{\pi R}{\alpha_5} + \frac{\Lambda}{2k}\frac{\tilde{b}}{2\pi}\left(\log\left(\frac{\Lambda}{\mu}\right)\right)^2, \tag{3.9}$$

where we relate $v \sim \Lambda$, $v_{N-1} \sim \mu$. Therefore, we obtain the result containing $\log^2(\Lambda/\mu)$ which looks weird. A similar result was obtained by Randall and Schwartz before doing the correct Greens function renormalization.

The second option looks more plausible. The boundary condition is

$$\frac{2\pi}{\alpha_1}|_v = \frac{2\pi}{\alpha_2}|_{v_1} = \cdots \frac{2\pi}{\alpha_N}|_{v_{N-1}} = \frac{2\pi}{\alpha}. \tag{3.10}$$

and the low-energy gauge couplings depends on the high-energy ones as:

$$\frac{1}{\alpha(M_Z)} = \frac{N}{\alpha} + (N-1)\frac{b}{2\pi}\log\left(\frac{1}{\varepsilon}\right) + \frac{b_0}{2\pi}\log\left(\frac{v\varepsilon^{N-1}}{M_Z}\right). \tag{3.11}$$

For large N, the 5D interpretation of this equation is:

$$\frac{1}{\alpha(\mu)} = \frac{\pi R}{\alpha_5} + \frac{b}{2\pi}\log\left(\frac{\Lambda}{\mu}\right). \tag{3.12}$$

This just shows the usual logarithmic running for the gauge couplings even though the deconstruction model mimics the gauge bosons propagating in 5D AdS_5 orbifold. Another feature which is visible here is that matter localized on the TeV brane does not contribute to the running above TeV since its contribution is included in b_0 but not in b.

Another way to summarize this approach is to notice that in deconstruction it is the coupling $\tilde{\alpha}$ defined as:

$$\frac{1}{\tilde{\alpha}(Q)} \equiv \frac{1}{\alpha_{(n)}(Q)} - \frac{1}{\alpha_1^{(i)}(v)} - \frac{1}{\alpha_2^{(i)}(v\varepsilon)} - \cdots - \frac{1}{\alpha_n^{(i)}(v\varepsilon^{n-1})}, \qquad v\varepsilon^{n-1} < Q < v\varepsilon^n \tag{3.13}$$

which is logarithmically sensitive to the difference of the scales:

$$\frac{1}{\tilde{\alpha}(Q)} = -\frac{b}{2\pi}\ln\left(\frac{Q}{v}\right) \tag{3.14}$$

and thus it should be used to study the physics of RG running in AdS_5.

The boundary condition given in eqs. (3.10) is derived from 5D gauge couplings defined at UV scale preserving the symmetry of AdS_5. Since the actual cutoff measured by 4D observer located at y is Λe^{-ky} for 5D cutoff Λ, the correct boundary condition necessary for 4D theory is

$$\int_0^{\pi R} dy \frac{1}{g^2(\Lambda e^{-ky})} \longrightarrow \sum_{i=1}^{N-1} \frac{1}{\alpha_i(v_{i-1})} \tag{3.15}$$

which is given in eqs. (3.10). The integrating out procedure defining the boundary condition (i.e. $\Lambda > M_{\mathrm{GUT}}$) is in figure 2. The symmetry breaking scale differs for different position of y, and the unusual boundary condition is very important in checking whether the unification really occurs. Figure 3 shows the region that we are considering in the paper. For $\Lambda < M_{\mathrm{GUT}}$, the process to reach the boundary condition preserving the symmetry of AdS_5 involves filling up the parts we have integrated out which is shown in figure 3.

3.1 5D interpretation

There are two features in warped gauge theories which are different compared to the flat case. For flat extra dimensions, the logarithmic running becomes power law as we raise up the energy scale since the effective number of Kaluza-Klein modes circulating in the loop increases. More precisely, below $1/R$ there is only one mode contributing to the one-loop calculation. At energies above the compactification scale the number of particles contributing to the running is determined by the number of KK modes with masses below the corresponding energy scale and for n extra dimension it has a simple scaling according to energy

$$N(E) \sim E^n\,. \tag{3.16}$$

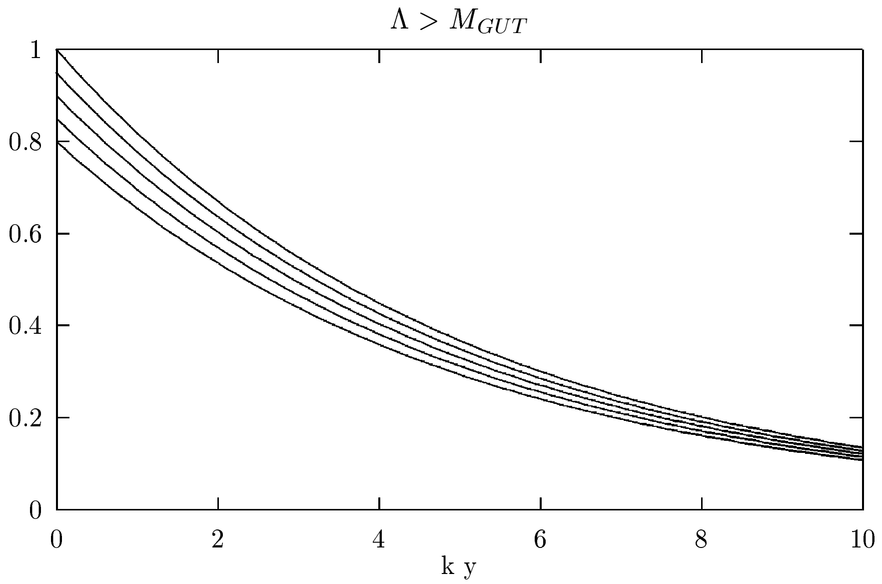

Figure 2: Wilsonian integrating out procedure above $M_{\rm GUT}$ which keeps the symmetry of AdS_5.

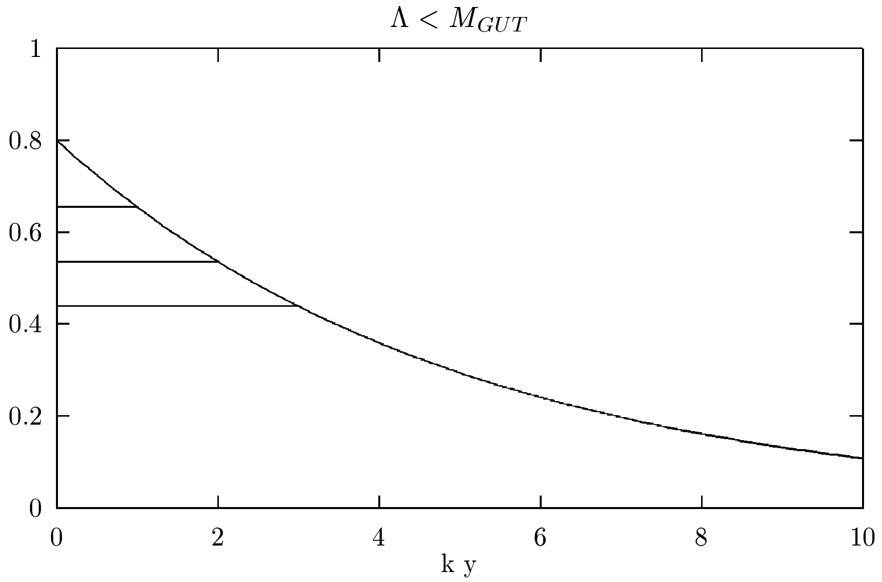

Figure 3: Wilsonian integrating out procedure below $M_{\rm GUT}$ which is based on 4D momentum.

The differential equation is given by

$$\frac{1}{\alpha(E+\Delta E)} = \frac{1}{\alpha(E)} + bN(E)\log\frac{(E+\Delta E)}{E}\,. \tag{3.17}$$

Integrating it from $1/R$ to Λ one obtains

$$\frac{1}{\alpha(\Lambda)} = \frac{1}{\alpha(1/R)} + c_n b(\Lambda R)^n + \cdots. \tag{3.18}$$

For example, we get a linearly cutoff dependent correction for the gauge theory with one extra dimension. But in this case the threshold correction at the cutoff scale is also given by $c\Lambda$. Unlike the case of the large logarithmic running in MSSM, the power law running effect is always comparable to the threshold correction.

In warped gauge theories, the first difference comes from the spectrum. The levels of Kaluza-Klein modes are not equal spaced and the mass difference between adjacent one becomes bigger for higher KK modes. Therefore the effective number of KK modes below certain energy E is

$$N(E) \sim \log E\,. \tag{3.19}$$

This softens the power running behavior by a certain amount but can not explain why one gets just a logarithmic running like in four-dimensional theory since the integration of the differential equation gives $(\log)^2$ rather than log.

$$\frac{1}{\alpha(\Lambda)} = \frac{1}{\alpha(1/R)} + \frac{1}{2} b \left(\log(\Lambda R)\right)^2 . \tag{3.20}$$

Here comes the second difference. In flat extra dimension, all the couplings of the KK modes are the same and are simply related to the higher dimensional coupling. In particular, the zero mode gauge coupling is related to the higher dimensional gauge coupling by the volume factor. By looking at the zero mode coupling, we can extract higher dimensional coupling. However, for the warped extra dimension, the coupling defined at the scale above TeV is not a zero mode coupling but a coupling localized near the Planck brane. Figure 2 shows the region in which the coupling is defined according to the energy scale above TeV. For extremely high energy scales near the Planck scale, the corresponding coupling is defined only at the Planck brane. The discrepancy between the arguments based on effective KK modes available at certain energy and the result obtained in the deconstruction lies here. Since the coupling we are looking at does not exist in the entire interval, it is very crucial to know what is the wave function overlap of each KK mode with the corresponding region. The interesting observation, which can be made in the deconstruction models, is that, though the effective number of KK modes is logarithmically increasing at high energies, the net contribution with the inclusion of the wave function overlap is always that of a single particle contribution. The coupling that we are interested in exists from $y = 0$ to y_* for given energy $E = \Lambda e^{-ky_*}$. As a specific example, let us see the energy range between the first and the second KK mode. At this scale we study the running of the coupling defined from $y = 0$ to $y_* = (\frac{N-1}{N})\pi R$ and only the zero mode and the first KK mode are light enough to contribute to the running. The zero mode is constant along the extra dimension and contributes $(\frac{N-1}{N})$ which is exactly proportional to the region on which the gauge coupling is defined. However, the first KK mode is localized in the region $(y_*, \pi R)$ and its contribution to the coupling in $(0, y_*)$ is just $(1/N)$. Therefore, the net contribution is that of single mode. Even at the highest energy scale we can reach (Λ), the net contribution is just that of one particle though all the KK modes are contributing to the running of the Planck brane localized gauge coupling. The gauge coupling is defined only near $y = 0$ and the zero mode has a wave function overlap $1/N$ with the region, the lightest one has $\frac{1}{(N-1)N}$, the second lightest one has $\frac{1}{(N-2)(N-1)}$, etc. and the sum is

$\frac{1}{N}+(-\frac{1}{N}+\frac{1}{N-1})+(-\frac{1}{N-1}+\frac{1}{N-2})+\cdots+(-\frac{1}{2}+1)=1$. It is a very interesting observation captured in the deconstruction. The logarithmic running obtained here is in accord with the result of Pomarol [4] and Randall-Schwartz [5] for $\Lambda \ll k$.

It should be kept in mind that the Kaluza-Klein mode analysis given here is just a convenient way of deriving RG equation in AdS_5 (more generally, in the curved space). Since UV parameters are defined at different scales for a 4D observer, it is not trivial to relate the UV parameter and the experimentally measured IR parameter. In solving these kinds of problems, the KK analysis via deconstruction is very useful and shows the physics transparently. However, this does not mean that we can derive physical spectrum above TeV.

3.2 Comment on the continuum limit

Since the low energy gauge coupling is given by that of the diagonal subgroup, it is not possible to raise N to be arbitrary large while keeping the gauge groups above the deconstruction scale weakly coupled. In the case of flat extra dimensions,

$$\frac{1}{\alpha}=\frac{N}{\alpha_i}\,, \tag{3.21}$$

and for the low energy coupling $\alpha \sim 1/20$, we get the maximum number of N to be 240 with $\alpha_i \sim 4\pi$. Similar restriction applies here. If we restrict our analysis to warped gauge theory with known hierarchy, the restriction on N is related to the upper bound on the cutoff compared to the curvature scale. With the notation given above,

$$\begin{aligned} \varepsilon^N &\sim 10^{-16}\,, \\ \varepsilon &= e^{-k/\Lambda}\,, \end{aligned} \tag{3.22}$$

and we get

$$\frac{k}{\Lambda}\geq\frac{1}{6}\,. \tag{3.23}$$

Thus $\Lambda = 6k$ is the maximum cutoff we can reach for the bulk Randall-Sundrum setup.[4] This is enough to study the interesting physics since Λ is high enough.

However, as we increase N, the neglected corrections become comparable to the log correction. In the derivation of the running equation for the gauge couplings, it is assumed that $\varepsilon \ll 1$ such that $\log(1/\varepsilon) \gg \mathrm{O}(1)$ at each matching scale. Once ε becomes close to one, the threshold correction can not be neglected. At the same time, the accumulated effect gets bigger as N increases. The power law correction $N\varepsilon^2$ becomes of order one for $N \sim 30$ with $\varepsilon \sim 1/5$. This is when $\Lambda \sim 1.6k$. For $\varepsilon \sim 1/5$, the logarithmic correction is already the same order as order one threshold correction. Therefore, the logarithmic correction derived here is spoiled by the power law correction unless $\varepsilon \ll 1$. Furthermore, there is a threshold correction at the curvature scale which might not be universal when the GUT breaking scale (Λ) is above the curvature scale.

[4] The bounds on Λ might be stronger if we consider $\Lambda > k$. In the derivation we used flat space constraint on α which is very similar for $\varepsilon \ll 1$. However, for $\varepsilon \sim 1$, power law correction appears and the restricted value of N might be smaller than 240.

For the unified theory living on a (flat) higher dimension, the threshold correction is not negligible and we can not make a clean prediction based on the low energy gauge couplings [20]. The same problem appears if we raise our cutoff (the GUT breaking scale) Λ to be higher than the curvature scale k.

4. Conclusions

In a gauge theory in the AdS_5 background, unlike in the flat case, the gauge coupling runs *logarithmically*. This observation may shed light on the possibility of having a unified theory in the Randall-Sundrum framework. In this paper we studied the running of gauge couplings in four-dimensional deconstruction models which mimic warped gauge theories. Deconstruction allows us to understand some properties of higher dimensional gauge theories with the help of the standard tools of four-dimensional gauge theories.

When the scale difference for different link-Higgs fields is large enough, $\varepsilon \ll 1$, which corresponds to the cutoff of the 5D theory lower than the curvature scale, the running is purely logarithmic. It might be more natural to keep the cut-off above the curvature scale so that one could define the theory at high-energies in an effectively flat background. But once we do this, the cutoff dependent threshold correction and the curvature dependent threshold correction ruin the predictability for the gauge unification since these unknown corrections are comparable to the calculable part. This gives us some insight on the features that a highly predictable GUT theory should possess.

Once the GUT breaking scale ($M_{\rm GUT}$) is below the curvature scale, then above $M_{\rm GUT}$ three gauge couplings get common corrections, and the different correction start to appear below $M_{\rm GUT}$. Then the result obtained in the paper gives a high precision prediction on the possibility of the unification regardless of the physics above $M_{\rm GUT}$ which is common to three gauge couplings.

Constructing realistic unification model of the warped gauge theory which can avoid proton decay remains as a future work.

Acknowledgments

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A. Bulk fields

In this appendix we investigate the effect of a replicated scalar field on the running of gauge couplings in the deconstruction set-up. Following the analysis of section 3 we will keep track of the evolution of the coupling $\tilde{\alpha}(Q)$ defined in eq. (3.13).

A complex 5D bulk field $h(x^\mu, y)$ in the fundamental representation is accounted for in decostruction by putting at each site a complex field $h_i(x^\mu)$ in the fundamental representation of the i-th group. Moreover, scalars living on neighbouring sites should communicate

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via the link-Higgs fields. In a non-supersymmetric set-up, the most general set of such couplings is very complicated so in the following we restrict ourselves to just one specific example when kinetic and interaction terms are:

$$\sum_{i=1}^{N}\left(\partial_\mu h_i^\dagger \partial_\mu h_i - \sum_{i=1}^{N-1}|g\Phi_i h_{i+1} - m_i h_i|^2\right). \tag{A.1}$$

In the continuum limit this lagrangian corresponds to:

$$\mathcal{L} = -\int dy\left(\partial_\mu h^\dagger \partial_\mu h - a^2(y)|\partial_5 h - M\epsilon(x_5)h|^2\right), \tag{A.2}$$

where $g_i v_i \sim a(y_i)/dy$ and $m_i - g_i v_i \sim a(y_i)M(y_i)$. Thus for a choice $v_i = v\varepsilon^i$, $m_i = xgv_i$ our deconstruction lagrangian (A.1) mimics the 5D bulk scalar in the RS background (appropriately rescaled so as to get the kanonical 4D kinetic term) with a constant kink-mass term

$$M \sim (x-1)gv\,. \tag{A.3}$$

Note that both lagrangian of eqs. (A.2) and (A.1) yield a massless mode. In the 5D case its profile is given by $h^{(0)} = \exp(M|y|)$ while in the deconstruction it is $h_i^{(0)} = x^i$. Since the matching (A.3) yields $x \sim 1 + M/(gv) \sim 1 + M\Delta y$ we see we can reliably mimic scalar with a kink-mass not greater than $(\Delta y)^{-1}$.

We are now ready to investigate the effect of such replicated scalar field on the running. We consider seperately three cases:

1. $x = 1$ (vanishing bulk mass term)

   Above $v\varepsilon$, $1/\tilde{\alpha}(Q)$ is defined as $\frac{1}{\alpha_{(1)}(Q)} - \frac{1}{\alpha_1(v)}$ thus only h_1 (which is effectively massless at this scale) contribute to its running.

   At $v\varepsilon$ two groups are effectively broken and both h_1 and h_2 can contribute to the running of $\frac{1}{\tilde{\alpha}(Q)} = \frac{1}{\alpha_{(2)}(Q)} - \frac{1}{\alpha_1(v)} - \frac{1}{\alpha_2(v\varepsilon)}$. The relevant mass terms are:

   $$\mathcal{L} = -(gv\varepsilon)^2|h_1 - h_2|^2\,. \tag{A.4}$$

   Diagonalizing the mass matrix, we find that (for $g \sim 1$) one eigenvalue is $2v\varepsilon$ thus the corresponding state is decoupled below $v\varepsilon$. The other eigenvalue is zero (or more precisely, less than $v\varepsilon^2$), hence we conclude that down to $v\varepsilon^2$ the bulk scalar gives the replicated of *one massless scalar* to the running of $\tilde{\alpha}(Q)$.

   Repeating this analysis at the consecutive thresholds we always find the same qualitative picture: there is one combination of the scalars h_p which is effectively massless and the remaining states are decoupled. Thus, in deconstruction picture, for a replicated scalar without bulk mass term only the zero-mode contributes to the running of α and the running is given by:

   $$\frac{1}{\tilde{\alpha}(Q)} = -\frac{b_S}{2\pi}\log\left(\frac{Q}{v}\right), \tag{A.5}$$

   where b_S is the beta-function coefficient of a massless scalar in the fundamental representation.

2. $x < 1$, (negative kink mass)

   In this case, the zero mode is localized towards the first site and thus it should contribute to the evolution of the coupling from high down to low scales, just like a field on the Planck brane. In fact, the analysis is very similar to the case $x = 1$ described in the preceding paragraphs. At each threshold there is one massless state which contributes to the running and the remaining states can be considered decoupled. Thus, also for $x < 1$, the running is given be eq. (A.5).

3. $x > 1$, (positive kink mass)

   In this case, the zero mode is localized towards the N-th site and thus it is not necessarily seen at high-energies. The analysis is more complicated and, in general, an exact formula for the running cannot be found.

   Above $v\varepsilon$, only h_1 can contribute to the running of α. Its mass is $xv\varepsilon$ thus it decouples before reaching the $v\varepsilon$ threshold. Thus, the running is given by:

$$\frac{1}{\alpha_{(1)}(v\varepsilon)} = \frac{1}{\alpha_1(v)} - \frac{b_S}{2\pi}\log\left(\frac{x\varepsilon v}{v}\right). \tag{A.6}$$

   In the range $(v\varepsilon, v\varepsilon^2)$ h_1 and h_2 can contribute to the running of $\tilde{\alpha}(Q)$ and their mass terms are:

$$\mathcal{L} = -(gv\varepsilon)^2|h_2 - xh_1|^2 - (gxv\varepsilon^2)^2|h_2|^2 \,. \tag{A.7}$$

   The greater eigenvalue is approximately $(x^2+1)vg\varepsilon$ and the corresponding state is decoupled. The lower eigenvalue is approximately $(x^2-1)gv\varepsilon^2$ and for $x > \sqrt{2}$ this state decoples before reaching $v\varepsilon^2$. Thus the running is:

$$\begin{aligned}\frac{1}{\alpha_{(2)}(v\varepsilon^2)} &= \frac{1}{\alpha_{(2)}(v\varepsilon)} - \frac{b_S}{2\pi}\log\left(\frac{(x^2-1)\varepsilon^2 v}{v\varepsilon}\right) \\ &= \frac{1}{\alpha_1(v)} + \frac{1}{\alpha_2(v\varepsilon)} - \frac{b_S}{2\pi}\log\left(\varepsilon^2\right) - \frac{b_S}{2\pi}\left(x(x^2-1)\right).\end{aligned} \tag{A.8}$$

   Repeating the same analysis for consecutive thresholds we obtain:

$$\frac{1}{\tilde{\alpha}(Q)} = -\frac{b_S}{2\pi}\log\left(\frac{Q}{v}\right) - \frac{b_S}{2\pi}\log\left(f\left(\frac{M}{v}, Q\right)\right). \tag{A.9}$$

   Compared to the two previous cases there appears a correction depending on some function f of the effective kink mass and the RG scale Q.

B. Three-site model

It is meaningful to look at the simplest model showing interesting physics. Since it is not possible to analyze N-site model in general exactly without numerical analysis, cascading procedure is used to study warped gauge theory. This requires enough amount of scale differences for VEVs of different sites and the flat limit can not be taken continuously within this analysis. For three sites, it is possible to diagonalize 3×3 matrix exactly and this allows us to see both the flat and warped gauge theory limit at the same time by taking the parameters appropriately. Therefore in this appendix we study three site model.[5] ¿From the general expression, we get

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1+\varepsilon^2 & -\varepsilon^2 \\ 0 & -\varepsilon^2 & \varepsilon^2 \end{pmatrix} \tag{B.1}$$

for $N = 3$. Three eigenvalues are

$$m_0^2 = 0\,, \tag{B.2}$$
$$m_1^2 = (1+\varepsilon^2) - \sqrt{1-\varepsilon^2+\varepsilon^4}\,, \tag{B.3}$$
$$m_2^2 = (1+\varepsilon^2) + \sqrt{1-\varepsilon^2+\varepsilon^4}\,. \tag{B.4}$$

Corresponding eigenstates are

$$A^{(0)} = \frac{1}{\sqrt{3}}(A^1 + A^2 + A^3)\,, \tag{B.5}$$
$$A^{(1)} = C(\varepsilon)(A^1 + (-\varepsilon^2 + \sqrt{1-\varepsilon^2+\varepsilon^4})A^2 + (-1+\varepsilon^2 - \sqrt{1-\varepsilon^2+\varepsilon^4})A^3)\,, \tag{B.6}$$
$$A^{(2)} = C'(\varepsilon)(A^1 + (-\varepsilon^2 - \sqrt{1-\varepsilon^2+\varepsilon^4})A^2 + (-1+\varepsilon^2 + \sqrt{1-\varepsilon^2+\varepsilon^4})A^3)\,, \tag{B.7}$$

where $C(\varepsilon)$ and $C'(\varepsilon)$ are the normalization coefficients.

Two interesting limits are

1. Gauge theory in flat space($\varepsilon = 1$)

   The eigenvalues are 3, 1 and 0 and the corresponding eigenstates are $(1, -2, 1)$, $(1, 0, -1)$ and $(1, 1, 1)$, respectively. This shows the dependence of KK mass on the variation of the coefficients. The heaviest mode has the biggest variation along the index (corresponding the coordinate along the extra dimension in the geometric interpretation).

2. Slightly warped gauge theory ($\varepsilon^2 = 1 - \delta$, $0 < \delta \ll 1$)

   This corresponds to the case in which $k \ll \Lambda$. The eigenvalues are $3 - \frac{3\delta}{2}$,$1 - \frac{\delta}{2}$ and 0 and the eigenstates are $(1+\frac{3\delta}{4}, -2, 1-\frac{3\delta}{4})$, $(1-\frac{\delta}{4}, \frac{\delta}{2}, -1-\frac{\delta}{4})$ and $(1, 1, 1)$, respectively. The eigenvalues are slightly smaller than the flat limit and the eigenstates start to show the tendency that the first excited mode has a more or less similar value for A_1 and A_2 while A_3 becomes bigger. For the heavest mode, A_3 part decreases and A_1 and A_2 becomes to be a similar magnitude with opposite sign.

3. warped gauge theory($\varepsilon \ll 1$)

   The eigenvalues are $2 + \frac{\varepsilon^2}{2}$, $\frac{3\varepsilon^2}{2}$ and 0 and the eigenstates are $(1, -1, 0)$, $(1, 1, -2)$ and $(1, 1, 1)$, respectively. The wave function shows the sharp peak at the position whose

[5] For two site model, it has only one scale for the VEV of link Higgs which always gives flat space geometry and warped geometry can not be seen.

local cutoff is similar to the Kaluza-Klein mass. The zero mode has a constant wave function. The first excited mode has a peak at the third site. The second (highest) excited mode has a peak at the second site.

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